

Locational Marginal Pricing (LMP)

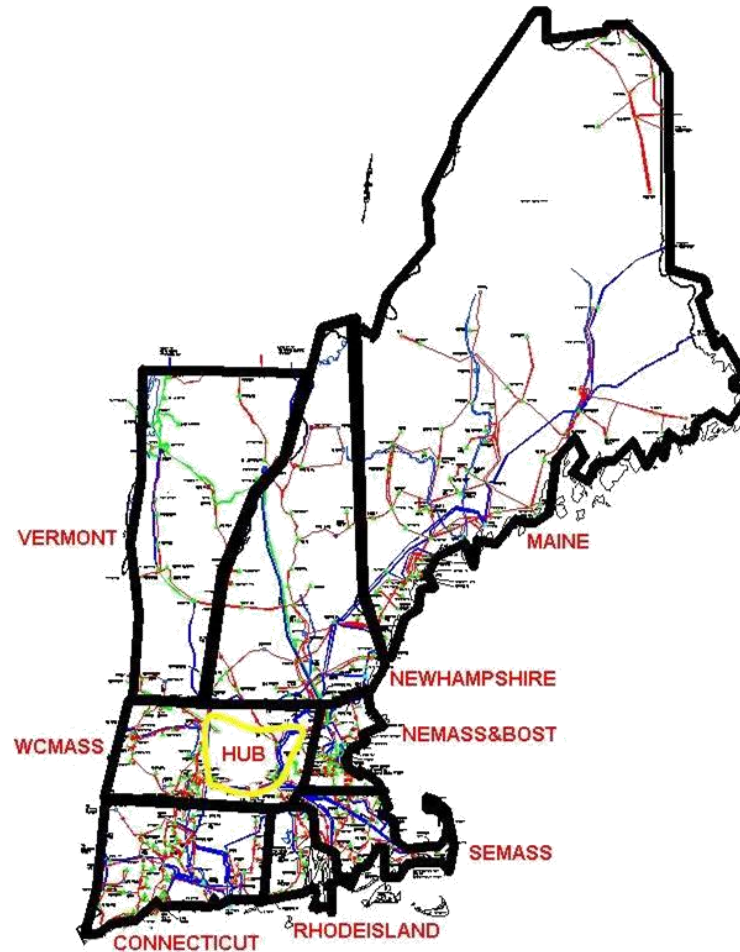
LMP Basics

Locations

Locations of LMPs

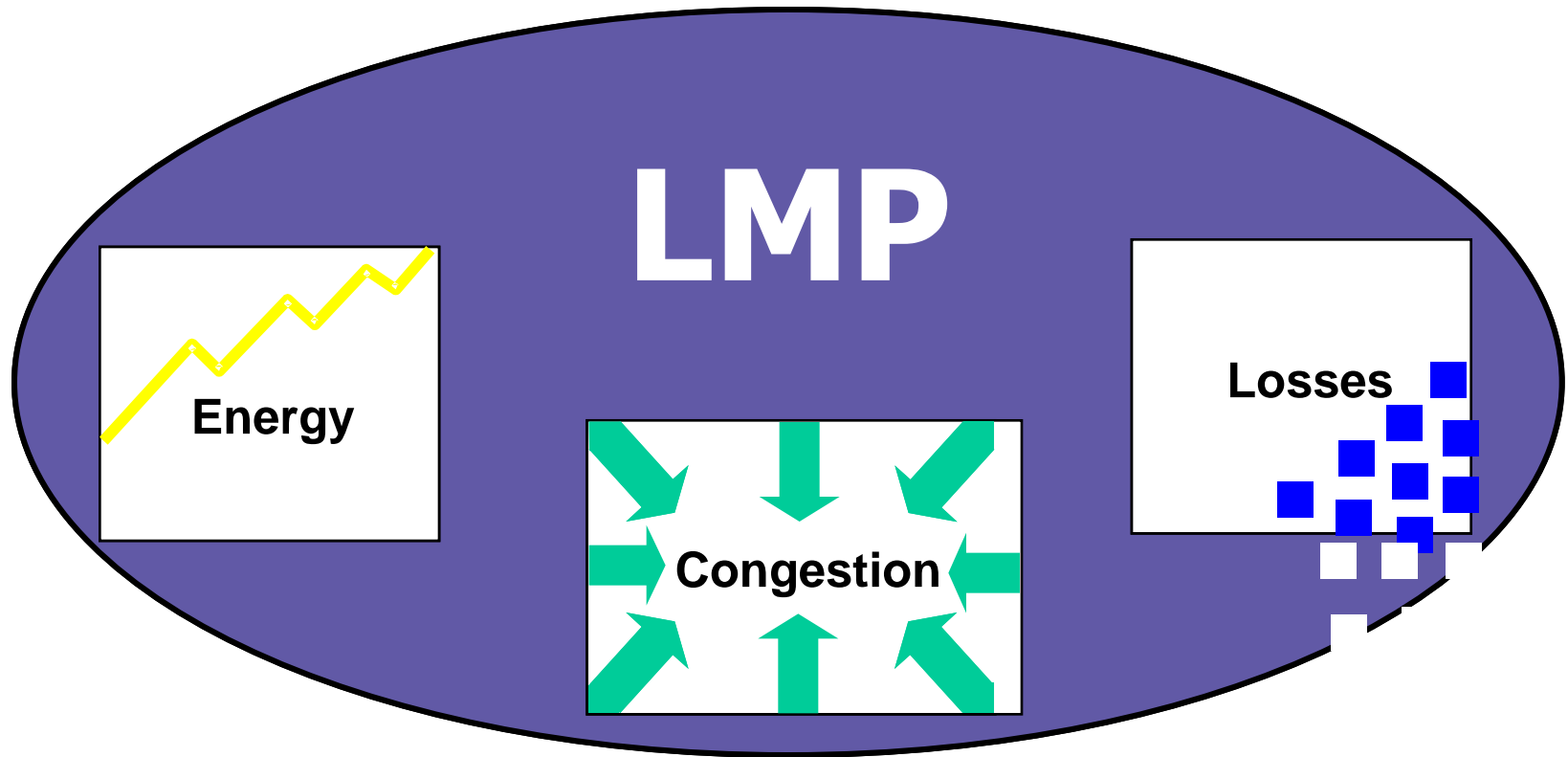
- **Node** – corresponds to a physical bus or collection of buses within the network
- **Load zone** – aggregation of nodes. Zonal price is the load-weighted average of the prices of all nodes in the zone.
- **Hub** – representative selection of nodes to facilitate long-term commercial energy trading. The hub price is a simple average of LMPs at all hub locations.
- **External/proxy node** – location that serves as a proxy for trading between ISO-NE area and its neighbors

NEPOOL Control Area and Pricing Hub



Constraints

LMP Components



Each LMP can be **split** into three components.

How Does Energy Get from Generator to Load?

- Two things have a major impact on flow:
 - Impedance
 - Resistance

How Does Energy Get from Generator to Load? (cont.)

- Think of it as two pipes:
 - one very narrow pipe (high impedance)



- One wider pipe (low impedance)



- Which can more volume of water flow through?

Location-based Marginal Price (LMP)

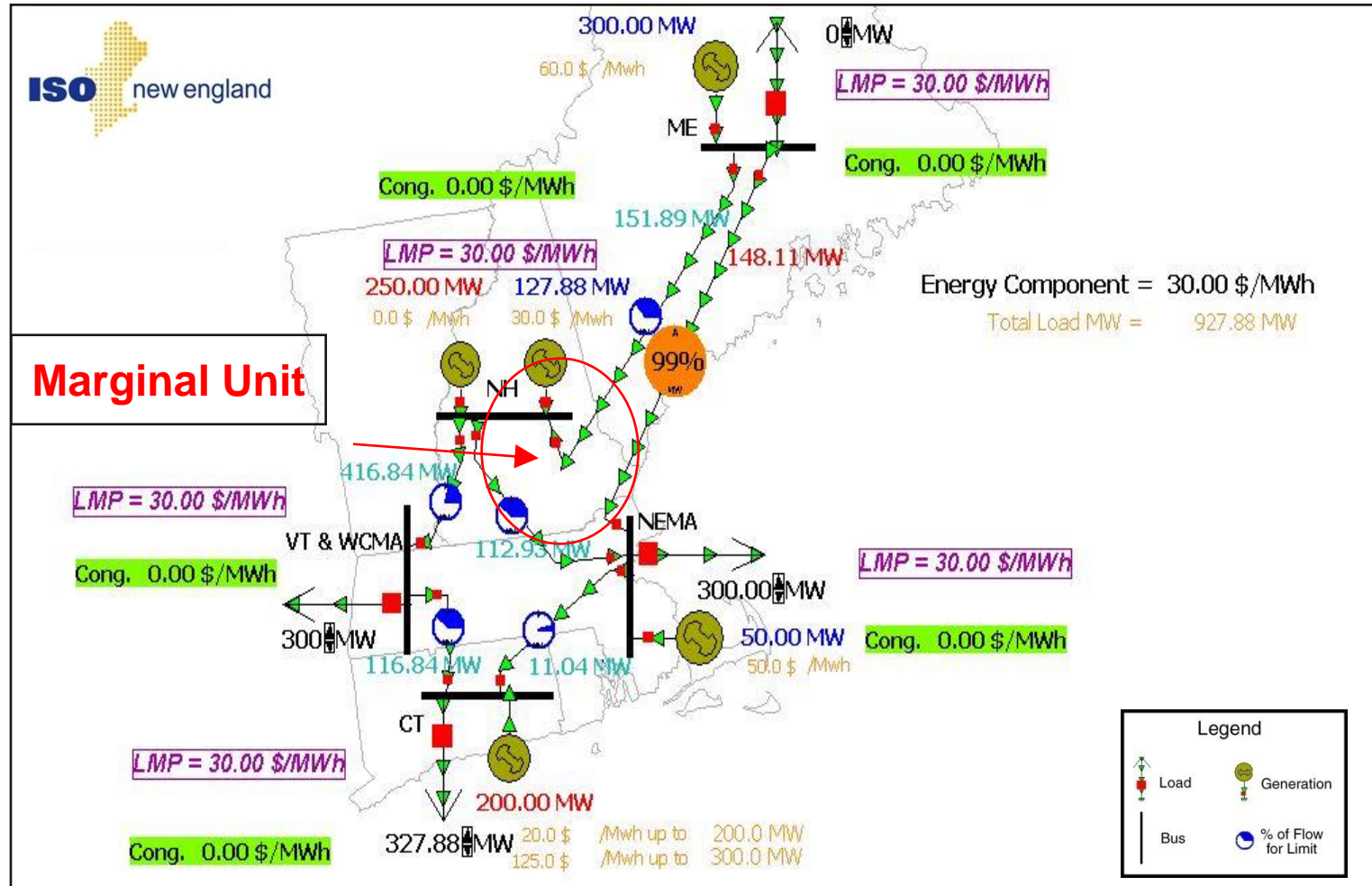
- LMP is the cost of supplying an increment of load at a particular location.
- One can think of the LMP as a change of the total production cost to deliver an additional increment of load to a location (We will show this in later examples), while respecting all constraints.
- LMPs are usually produced as a result of economic dispatch.
- LMPs can be calculated looking ahead – ex-ante LMPs or after-the-fact – ex post LMPs. Ex-ante LMPs for generation locations are also called Nodal Dispatch Rates (NDR).

Calculation of LMPs

- LMPs are calculated for all locations:
 - Day-Ahead Market (DAM)
 - Hourly basis
 - Real-Time Market (RTM)
 - Every five (5) minutes during the Operating Day

Reference Market Rule 1, Section 2.1.

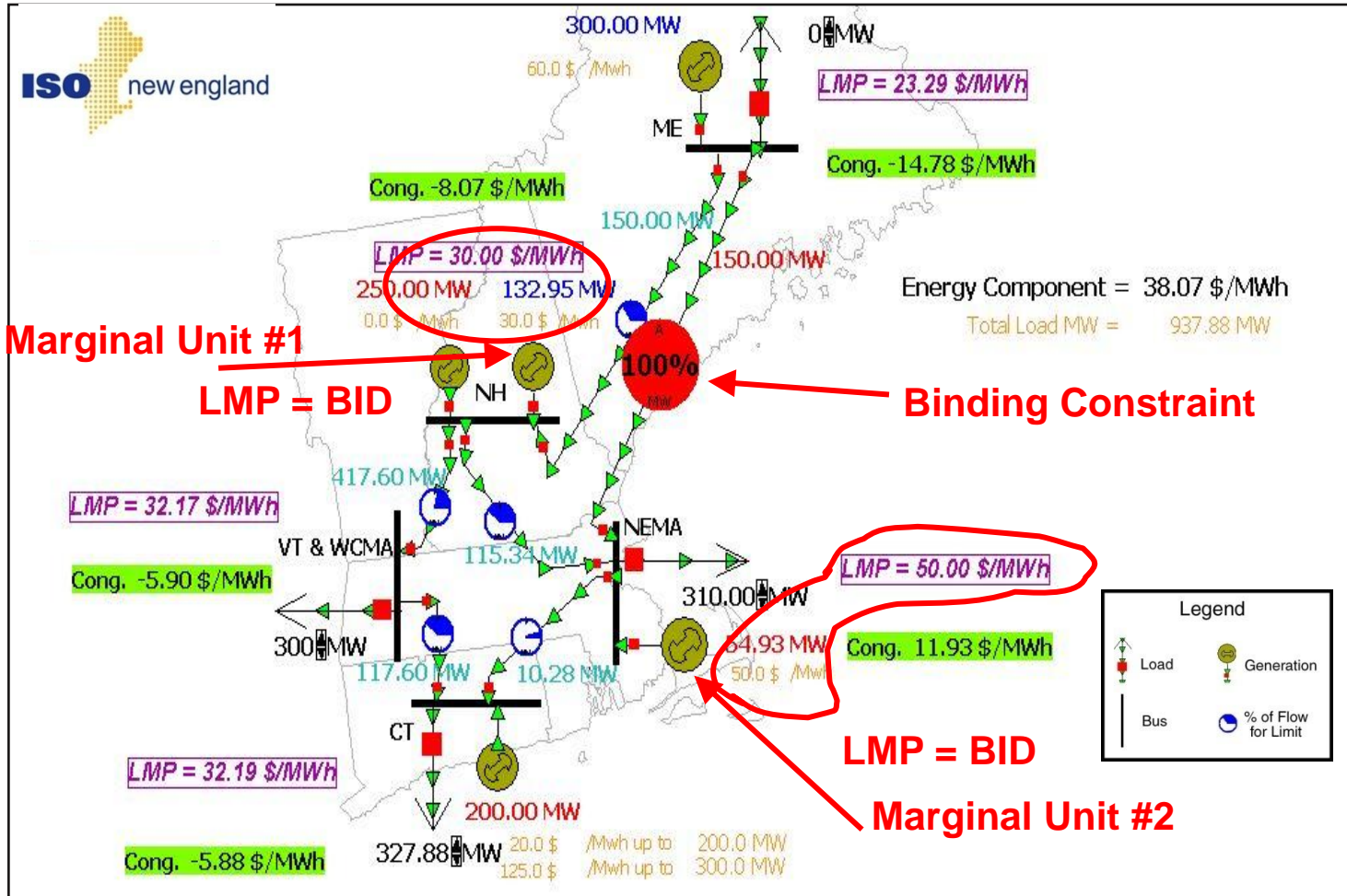
System With No Constraints



LMP Calculation – Fundamental Properties

- The price at the location of each marginal unit is always equal to its offer price.
- $n+1$ Rule: For n binding constraints, there is at least $n+1$ marginal units.
- In the case of no congestion, there is only one marginal unit.

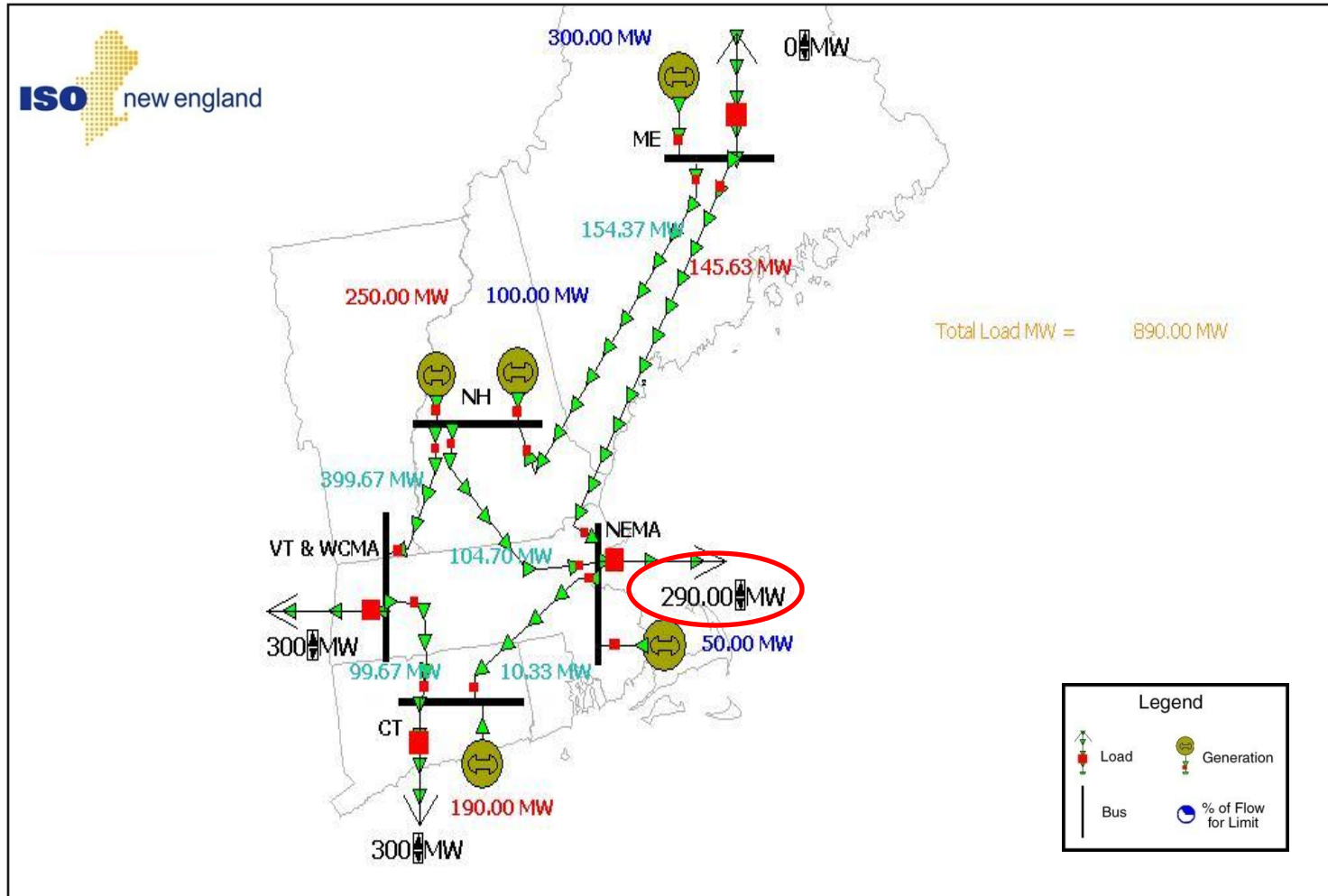
System with a Binding Constraint



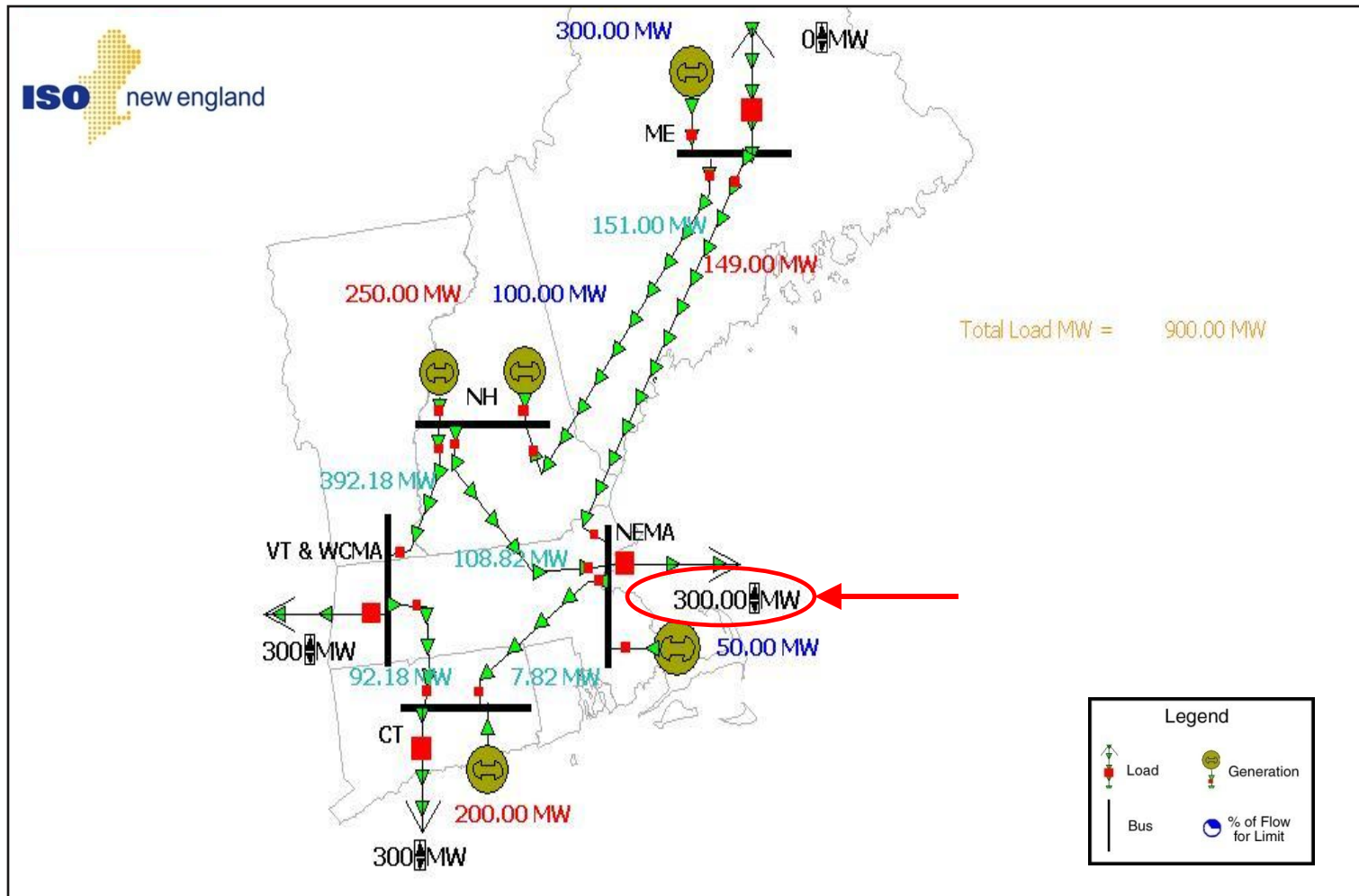
LMP Calculator

- Binding Transmission constraints converted to Sensitivities, one type of which is Power Transfer Distribution Factors (PTDF)
- PTDF determines a change in the power flow at each line when one (1) (or more) MW is transferred from one bus of the network to another.
- When a MW is transferred from one bus to another, it affects every single flow in the network.

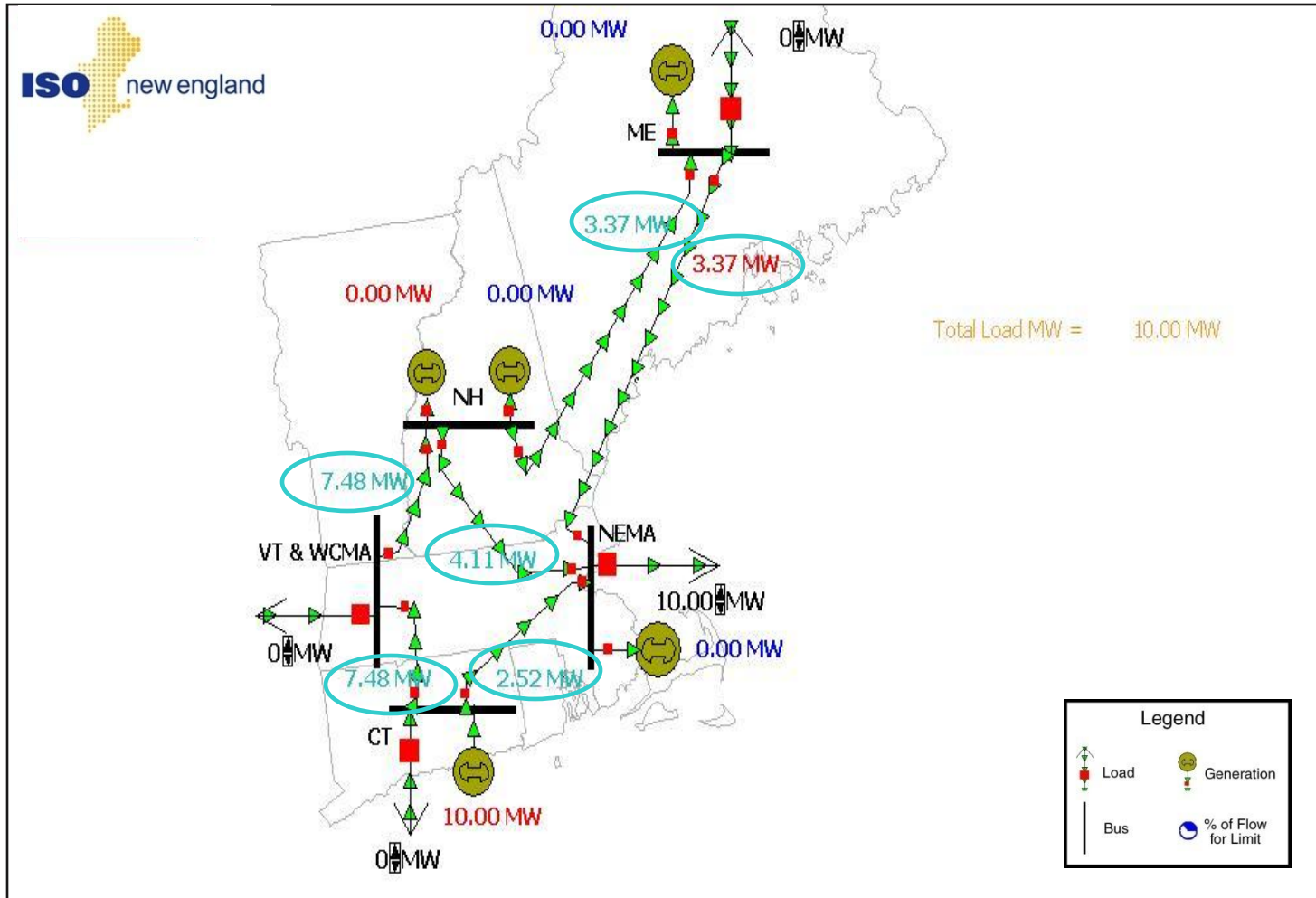
Power Transfer Distribution Factors (PTDF) – Example



PTDF – Example (cont.)



PTDF – Example (Differences in Flows)

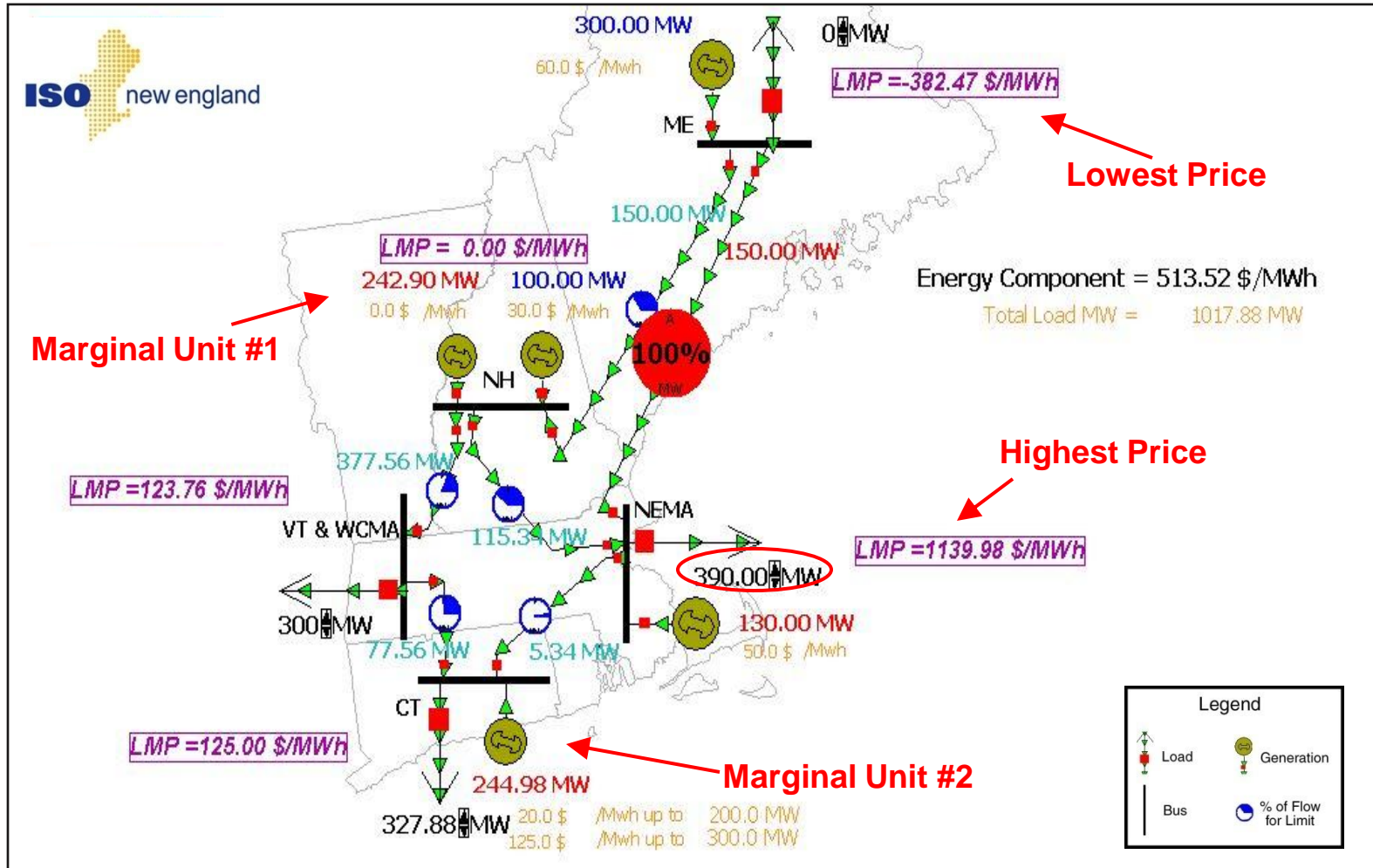


PTDF

Line Records				
From Number	From Name	To Number	To Name	MW
1	NH	2	VT & WCMA	-7.483
1	NH	4	NEMA	4.113
1	NH	5	ME	3.370
2	VT & WCMA	3	CT	-7.483
3	CT	4	NEMA	2.517
5	ME	4	NEMA	3.370

Line PTDF Records						
From Number	From Name	To Number	To Name	Circuit	From % PTDF	To % PTDF
1	NH	2	VT & WCMA	1	-74.83	74.83
1	NH	4	NEMA	1	41.13	-41.13
1	NH	5	ME	1	33.7	-33.7
2	VT & WCMA	3	CT	1	-74.83	74.83
3	CT	4	NEMA	1	25.17	-25.17
5	ME	4	NEMA	1	33.7	-33.7

High and Negative LMP Prices



Using PTFDs to Calculate the Prices

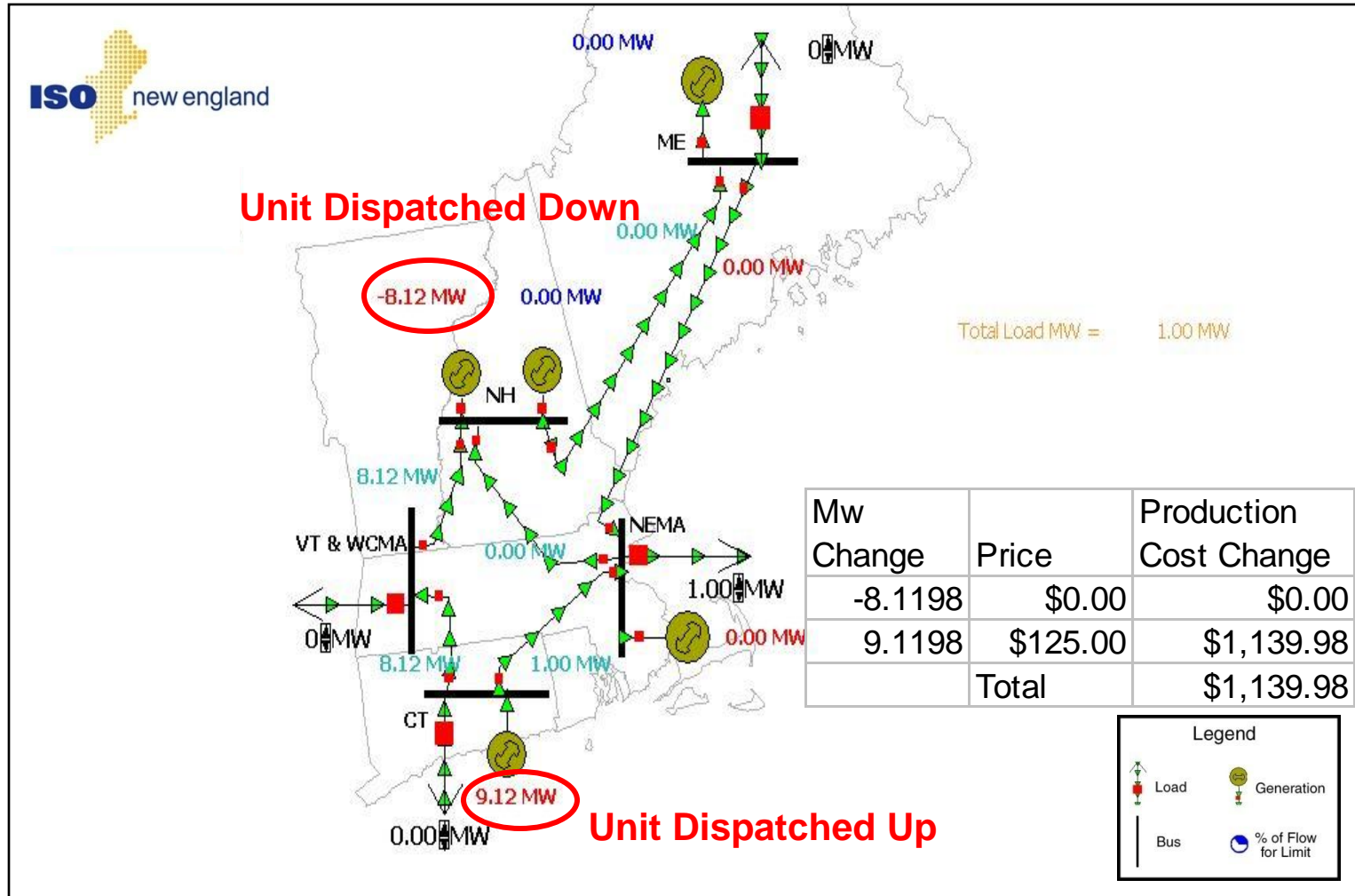
Line PTFD Records		(Delivery from NH to NEMA)					
From Number	From Name	To Number	To Name	Circuit	From % PTFD	To % PTFD	
1	NH	2	VT & WCM	1	15.02	-15.02	
1	NH	4	NEMA	1	47.7	-47.7	
1	NH	5	ME	1	37.28	-37.28	
2	VT & WCMA	3	CT	1	15.02	-15.02	
3	CT	4	NEMA	1	15.02	-15.02	
5	ME	4	NEMA	1	37.28	-37.28	Constraint

Line PTFD Records		(Delivery from CT to NEMA)					
From Number	From Name	To Number	To Name	Circuit	From % PTFD	To % PTFD	
1	NH	2	VT & WCM	1	-75.66	75.66	
1	NH	4	NEMA	1	42.47	-42.47	
1	NH	5	ME	1	33.2	-33.2	
2	VT & WCMA	3	CT	1	-75.66	75.66	
3	CT	4	NEMA	1	24.34	-24.34	
5	ME	4	NEMA	1	33.2	-33.2	Constraint

Simple Arithmetic

- Need to serve one MWh @ NEMA
- 37% of one (1) MWh from NH will flow over constrained line.
- 33% of one (1) MWh from CT will flow over constrained line.
- This means $(\text{NH reduction} * -37\%) + (\text{CT Increase} * 33\%)$ must = 0 since the flow over the constrained line cannot be increased and NH change must be 1 less than CT increase.

Determining How the High Price was Derived



PTDF to Calculate Price

- PTDF can determine the MW dispatch to solve the additional increment.
- So PTDF can be used to calculate LMP by using Prices in the calculation.

Unit	MW Change	Cost Change (\$/MW * MW Change)	PTDF	Change Flow Over ME-NEMA Path
NH	-8.12	0.00	0.37284	-3.03
CT	9.12	1139.97	0.33196	3.03
	1.00	1139.97	Change in MW vs. Change in Flow	0.00



FTR Basics

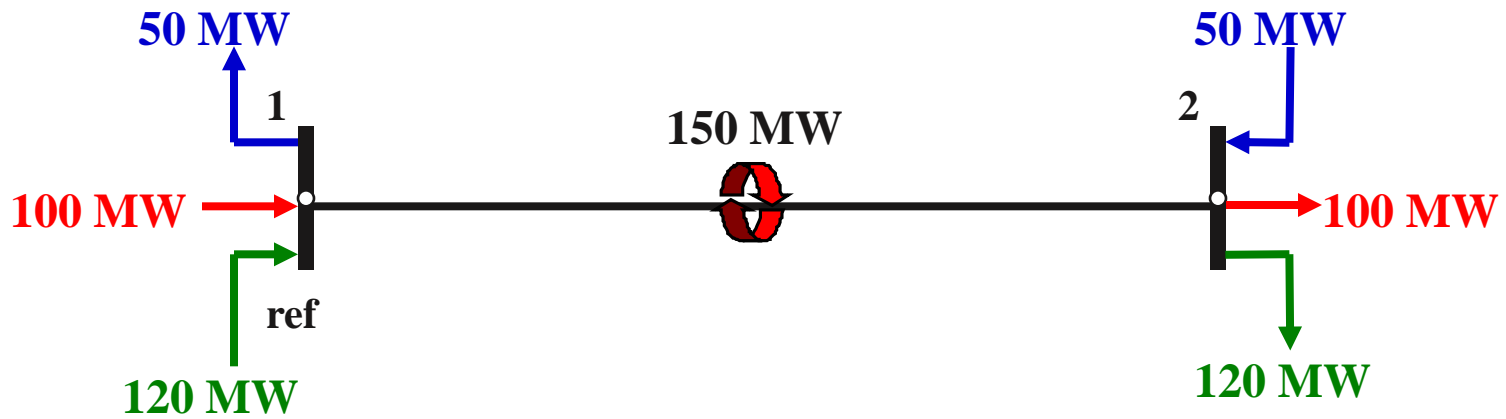
FTR Auction

- FTR Auction is a process of clearing FTR bids to buy and offers to sell in such a way that maximizes the auction value
- The objective of the auction is to award FTRs to those who value them the most, i.e., total value of the FTRs.
The FTR auction is not intended to maximize auction revenue!
- The FTR auction must respect all transmission constraints on the system – all awarded FTRs must be simultaneously feasible. In other words, ISO cannot oversell transmission capacity.

FTR Auction

- The auction accepts bids to buy FTRs and offers to sell them
- FTR auctions are held on annual and monthly basis
- At each monthly auction, the awarded annual FTRs will be considered as fixed
- Each bid and offer specifies the amount of MW F and the price C . This means that the bidder is ready to pay up to C \$/MW for transferring each F MW from source to sink

FTR Auction Two Bus Example



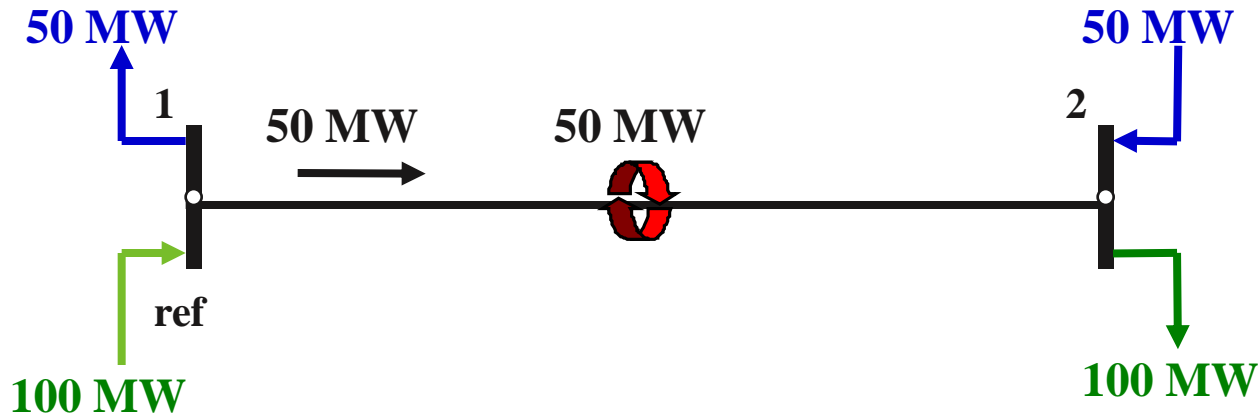
Bid F_1 : $F_1^{\max} = 120$ MW, $C_1 = \$20$.

Offer F_2 : $|F_2^{\max}| = 50$ MW, $C_2 = \$5$.

Fixed $F_{fix} = 100$ MW.

Sensitivities: $S_1 = 0$, $S_2 = -1$, $\alpha_{12} = S_1 - S_2 = 1$.

FTR Auction Solution – Two Bus Example



$$\text{Total Auction Value: } (20 * 100) - (5 * 50) = \$1750$$

- Let the transfer limit increase by 1 MW. We can sell now up to 101 MW FTR to the bidder. The auction value in this case will be $(20 * 101) - (5 * 50) = \1770 .
- This change in cost $\mu = \$1770 - \$1750 = \$20$ is a shadow price of the transmission constraint
- Again, unlike in Economic Dispatch, the shadow price is always positive

Two Bus Example – Clearing Price

- Let's calculate the change in the auction value to transfer 1 MW transaction
- In order to keep the flow on binding constraint unchanged, we will have to decrease F_1 by 1 MW
- At the cost of \$20/MW, the auction value will decrease by \$20.
- That will create the clearing price of \$20/MW for this path

FCP Calculation

- After clearing, all FTR bids and offers will end up in one of three groups:
 - At the maximum limit
 - At zero
 - Between zero and maximum
- The third group of FTRs is called Marginal FTRs – these are the FTRs that determine FCPs for **ALL** paths in the network

FTR – Fundamental Properties

- If an FTR bid is cleared in full, the FCP must be lower than its bid price
- If an FTR bid is not cleared, the FCP must be higher than its bid price
- If an FTR bid is cleared between 0 and max offer amount, the FCP must be equal to its bid price

FTR – Fundamental Properties (cont.)

- If an FTR offer is cleared in full, the FCP must be higher than its offer price
- If an FTR offer is not cleared, the FCP must be lower than its offer price
- If an FTR offer is cleared between 0 and max bid amount, the FCP must be equal to its offer price

FTR – Fundamental Properties (cont.)

- Each possible path in the network has its own clearing price
- The FCP for the marginal path always equals its bid or offer price
- For n binding constraints there are at least n marginal FTRs

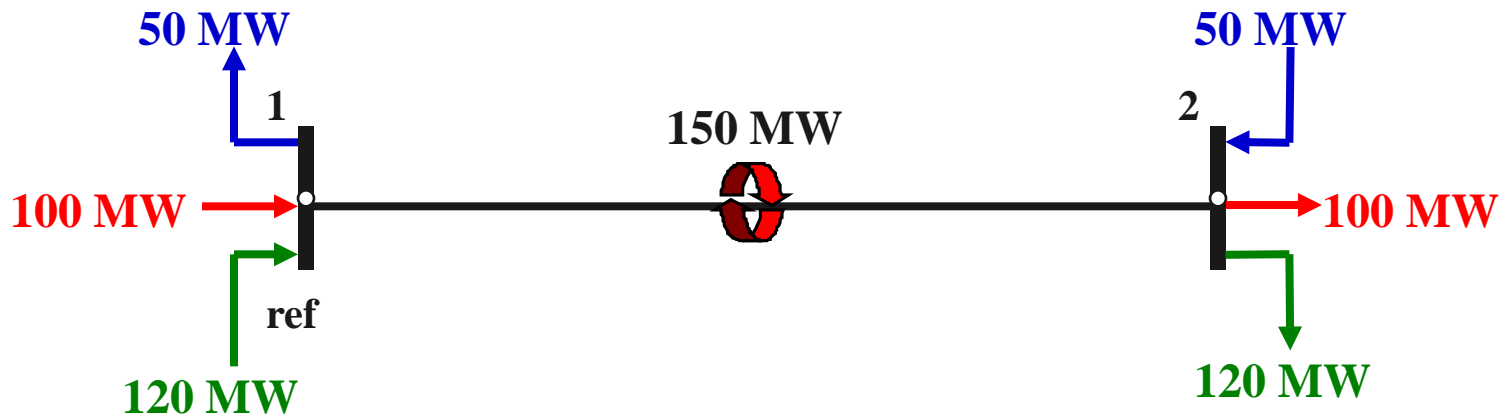
Negative FTR Bid

- Negative FTR bid is an FTR with negative price and positive MW
- This means that this FTR “adds” flow to the path in the positive direction
- It is mathematically equivalent to the FTR offer submitted in opposite direction (i.e., negative bid for $FTR_{ab} \Leftrightarrow FTR_{ba}$ offer)

Negative FTR Bid (cont.)

- One can consider negative FTR bid as a virtual FTR offer in the opposite to the bid direction. With this, anybody can submit an offer without having an FTR to sell
- The submitter of this bid **will be paid** in the auction and then will be charged in DA

Two Bus Example with the Negative Bid



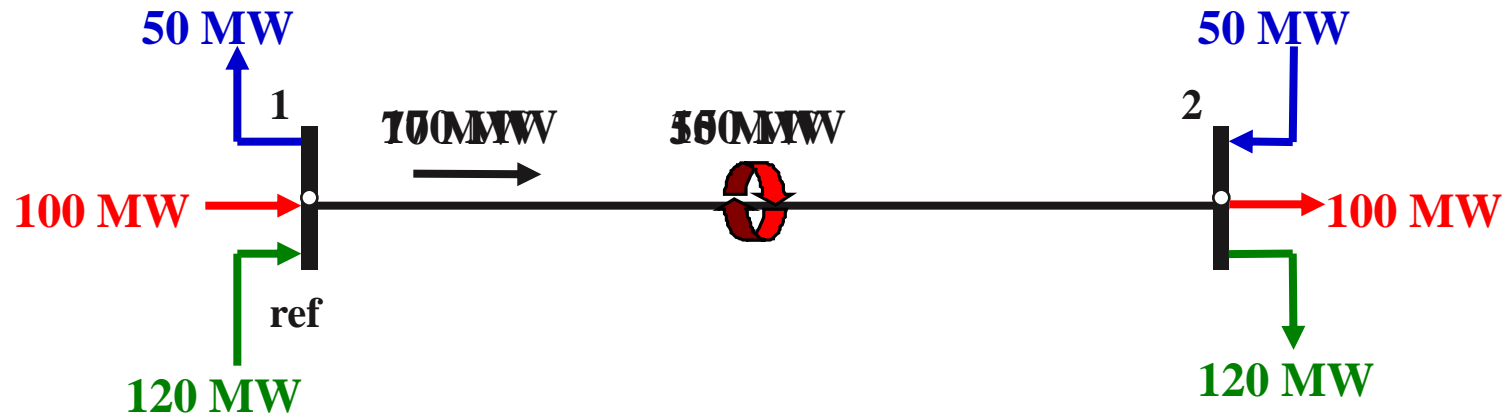
Bid F_1 ($1 \rightarrow 2$): $F_1^{\max} = 120$ MW, $C_1 = \$20$.

Bid F_2 ($2 \rightarrow 1$): $F_2^{\max} = 50$ MW, $C_2 = -\$5$.

Fixed $F_{fix} = 100$ MW.

Sensitivities: $S_1 = 0$, $S_2 = -1$, $\alpha_{12} = -\alpha_{21} = S_1 - S_2 = 1$.

Two Bus Example with Negative Bid (cont.)



Auction Value: $C_1 \cdot F_1 + C_2 \cdot F_2 = 20 \cdot F_1 + (-5) \cdot F_2$ - max

S.T.

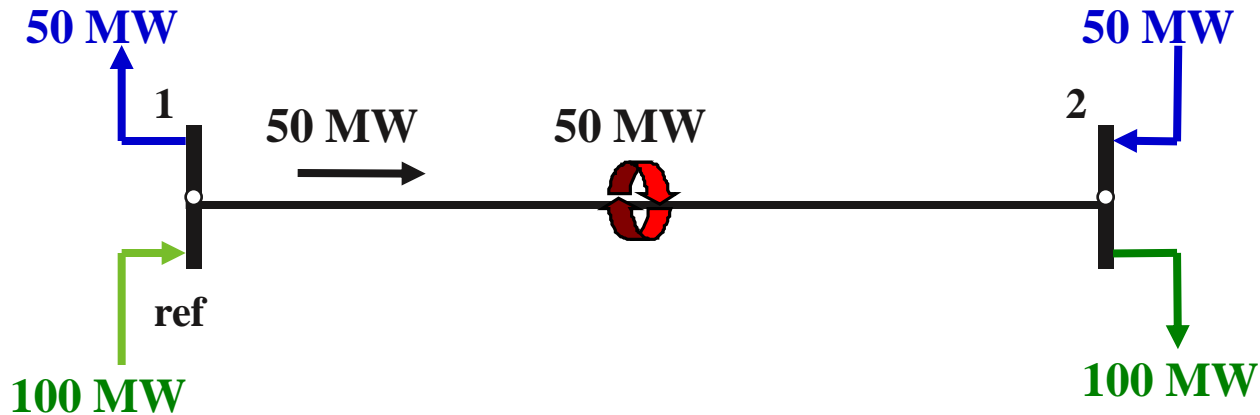
$$\alpha_{12} \cdot F_1 + \alpha_{21} \cdot F_2 + \alpha_{fix} \cdot F_{fix} \leq 150 \Rightarrow$$

$$1 \cdot F_1 + (-1) \cdot F_2 + 1 \cdot 100 \leq 150 \Rightarrow$$

$$F_1 - F_2 \leq 50 \text{ - transmission constraint}$$

$$0 \leq F_1 \leq 120; 0 \leq F_2 \leq 50 \text{ - capacity constraints.}$$

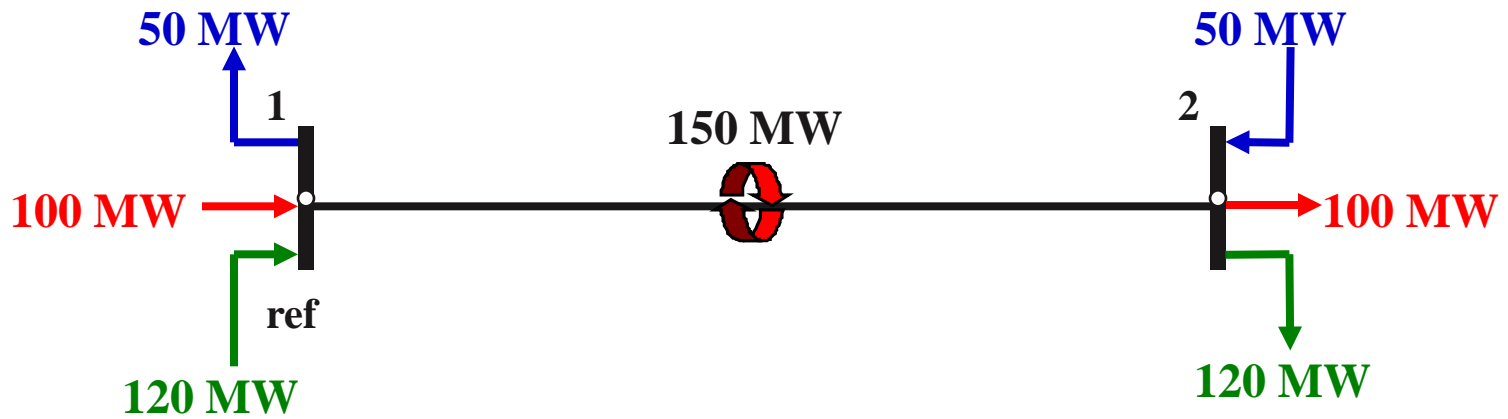
Solution: Two Bus Example with Negative Bid



$$\text{Total Action Value: } (20 * 100) + (-5 * 50) = \$1750$$

- $FCP_{12} = \$20$
- F_1 will be charged $20 * 100$ or \$2,000.
- F_2 will be charged $(-20) * 50$ or -\$1,000.
- The total auction revenue is $2000 - 1000$ or \$1000.

Two Bus Example with Negative Offer



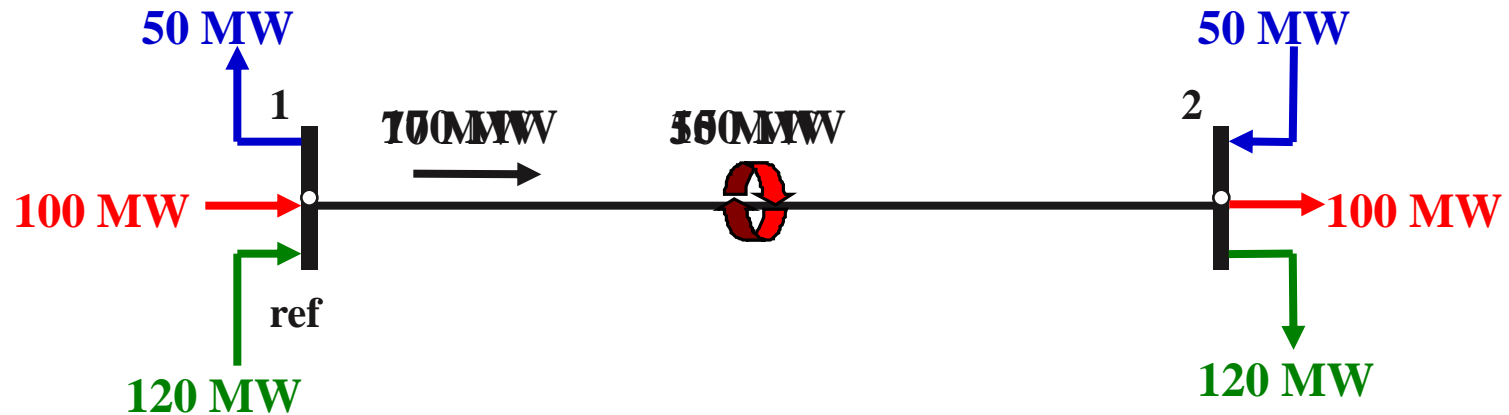
Offer F_1 ($2 \rightarrow 1$): $F_1^{\max} = 120$ MW, $C_1 = -\$20$.

Offer F_2 ($1 \rightarrow 2$): $F_2^{\max} = 50$ MW, $C_2 = \$5$.

Fixed $F_{fix} = 100$ MW.

Sensitivities: $S_1 = 0$, $S_2 = -1$, $\alpha_{12} = -\alpha_{21} = S_1 - S_2 = 1$.

Two Bus Example with Negative Offer (cont.)



Auction Value: $C_1 \cdot F_1 + C_2 \cdot F_2 = -20 \cdot F_1 + 5 \cdot F_2$ - max
 S.T.

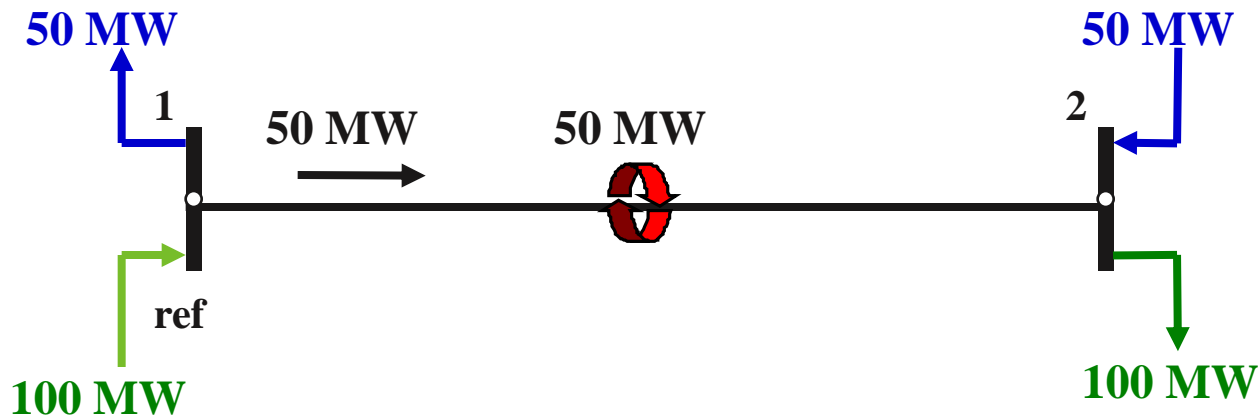
$$\alpha_{21} \cdot F_1 + \alpha_{12} \cdot F_2 + \alpha_{12} \cdot F_{fix} \leq 150 \Rightarrow$$

$$-1 \cdot F_1 + 1 \cdot F_2 + 1 \cdot 100 \leq 150 \Rightarrow$$

$$-F_1 + F_2 \leq 50 \text{ - transmission constraint}$$

$$-120 \leq F_1 \leq 0; \quad -50 \leq F_2 \leq 0 \text{ - capacity constraints.}$$

Solution: Two Bus Example with Negative Offer



Total Action Value: $(-20 * -100) + (5 * -50) = \1750

- $FCP_{12} = \$20$
- F_1 will be charged $(-20) * (-100)$ or \$2,000.
- F_2 will be charged $20 * (-50)$ or -\$1,000.
- The total auction revenue is $2000 - 1000$ or \$1000.

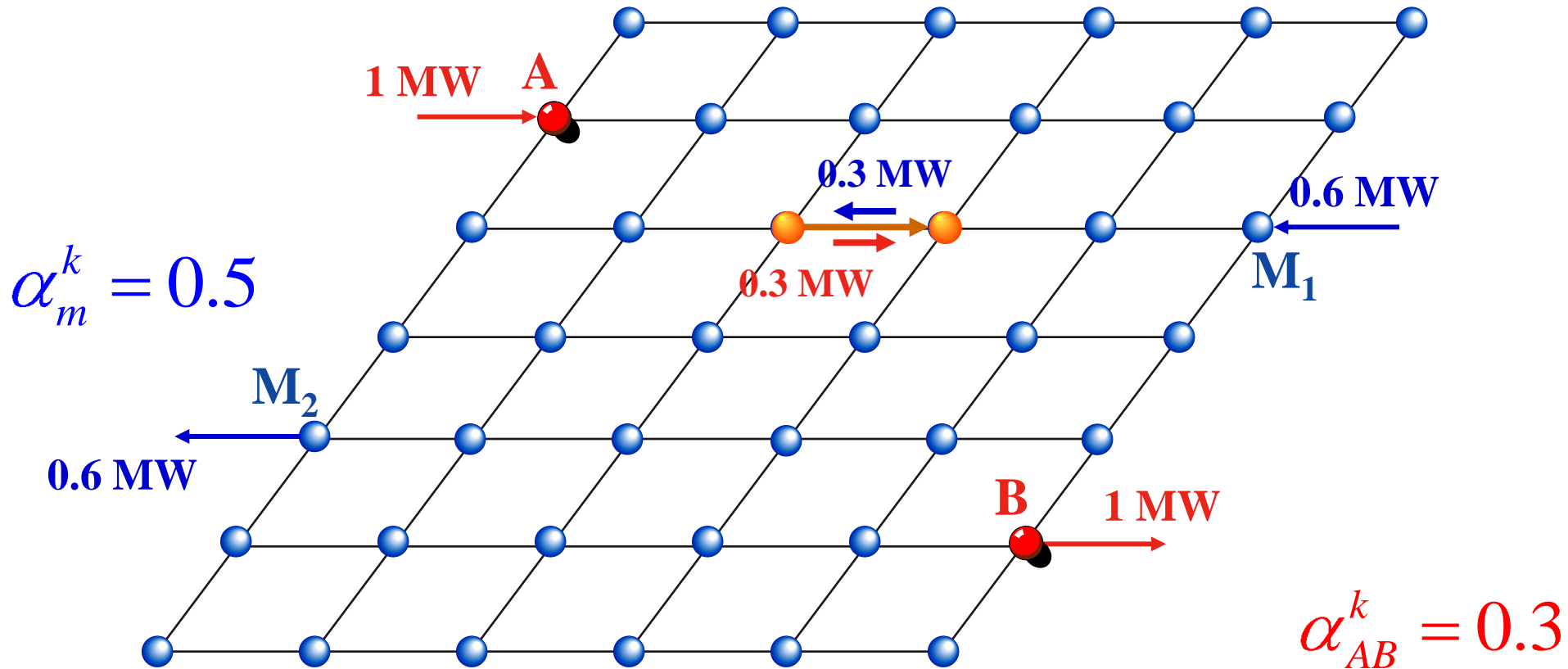
FTR Clearing Price

- The FTR clearing price for a path can be obtained considering the effect of a 1 MW transaction over that path
- The meaning of the clearing price is a cost of that additional transfer in terms of auction value
- If there are binding constraints, the effect of this additional transfer on the binding constraint should be compensated by some other transaction creating a counter flow (otherwise the limit will be violated.) Much like in the generators re-dispatch, this is achieved by adjusting marginal FTRs.

FTR Clearing Price (cont.)

- Due to the fact that each non-zero shadow price is determined by the marginal FTRs, each FCP is a function of all marginal FTRs
- FCP can be negative
- FCP can be used to determine why certain FTR bids or offers have not been cleared in the auction

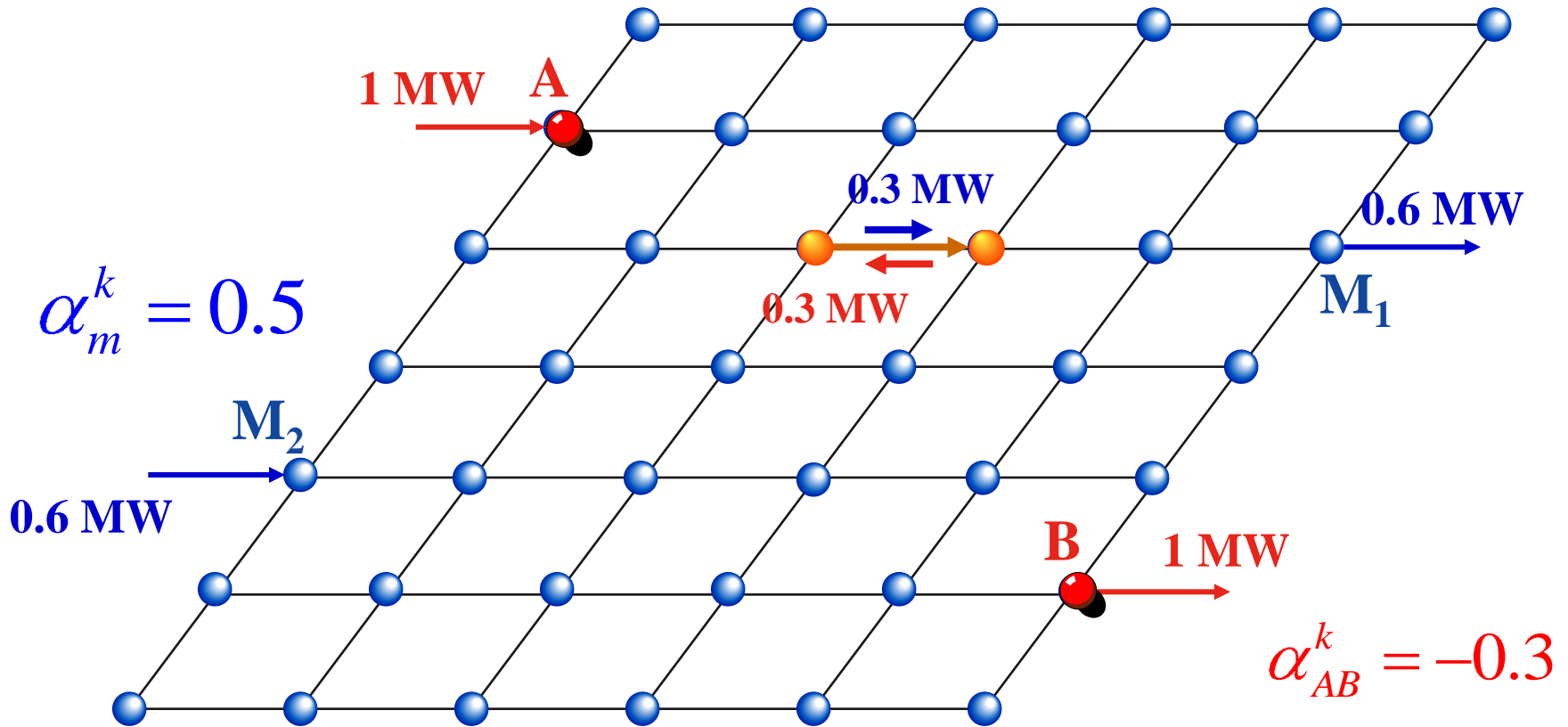
FTR Clearing Price Calculation Example



Change in cost: $-0.6 * 20 = -\$12 \rightarrow FCP_{AB} = \12 .
 $FCP_M = \$20 = 0.5 * \mu \rightarrow \mu = \40 .

Using FCP formula: $FCP_{AB} = 0.3 * 40 = \$12$.

FTR Clearing Price Calculation Example (cont.)



Change in cost: $0.6 * 20 = \$12 \rightarrow FCP_{AB} = -\12 .

Using FCP formula: $FCP_{AB} = -0.3 * 40 = -\12 .